Spin-charge separation in ultra-cold quantum gases

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We investigate the physical properties of quasi-1D quantum gases of fermion atoms confined in harmonic traps. Using the fact that for a homogeneous gas, the low energy properties are exactly described by a Luttinger model, we analyze the nature and manifestations of the spin-charge separation. Finally we discuss the necessary physical conditions and experimental limitations confronting possible experimental implementations.

One dimensional (1D) quantum liquids are very rich and interesting systems. In spite of their apparent conceptual simplicity, both the ground state and the excitations exhibit strong correlation effects and posses a number of exotic properties, ranging from spin-charge separation to fractional statistics (see [6, 7, 8] and ref. therein). Progress in creating, manipulating and studying ultra-cold quantum gases with controlled and adjustable interactions [1, 9], and in particular the recent development of 1D magnetic and optical wave guides opens the door for a new and clean physical realization of such 1D systems with the tools of atomic physics and quantum optics. While most of the recent theoretical and experimental work has focused on 1D Bose gases (as a Tonks gas or a quasicondensate) [2, 3] progress in cooling Fermi gases into the quantum degenerate regime [4] point to the possibility of realizing a Luttinger Liquid (LL) [7] with cold fermionic atoms. One of the key predictions of the Tomonaga-Luttinger model (LM) for interacting fermions is spin-charge separation [7]. It is a feature of interacting spin-1/2 particles and manifests itself in complete separation in the dynamics of spin and density waves. Both branches of the excitations are sound-like and characterized by different propagation velocities. This phenomena is one of hallmarks of a Luttinger liquid, however it has never been been demonstrated in a clean way in an actual condensed matter system (see e.g. [11, 12]). It is the purpose of this Letter to analyze in detail the conditions of realizing an (inhomogeneous) LL with a gas of cold fermionic atoms in 1D harmonic trap geometries, and in particular to study the possibilities of seeing spin-charge separation in the spectroscopy and wave packet dynamics of laser excited 1D Fermi gases.

The simplest example of a Luttinger liquid made of a gas of cold atoms consists of fermionic atoms with two ground states representing a spin-1/2 under quasi-1D trapping conditions. We assume the atoms to be cooled below the Fermi-degeneracy temperature $k_B T_F \sim N\hbar\omega$, where N is the number of particles and ω is the frequency of the longitudinal confinement. The condition for a quasi-1D system is tight transverse trapping in an external potential with the frequency $\omega_\perp$ exceeding the characteristic energy scale of the longitudinal motion. Due to the quantum degeneracy the longitudinal motion has all the energy levels up to the Fermi-energy $\epsilon_F \sim k_B T_F$ filled. Thus we require the total number of particles to
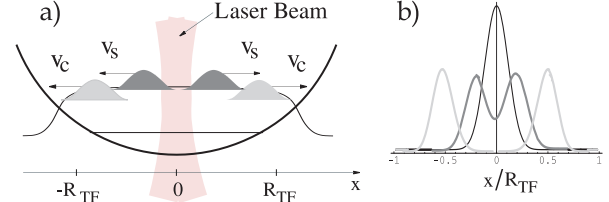

Fig. 1. (a) Schematic setup: a two component fermi gas is trapped in a harmonic potential in a 1D configuration. At time $t = 0$ a short laser pulse focused near the center of the trap excites (i) a density (charge) or (ii) a spin-wavepacket. (b) Wave packet dynamics for different times as function of position (in units of the Thomas Fermi radius R_{TF}): spin-charge separation manifests itself in a spatial separation of the spin (solid line) and density (dashed line) wave packets (shown at half a trap oscillation period $\omega t = 0.5$), which can be probed by a second short laser pulse at a later time. The parameters correspond to $N = 10^3$ ^{6}Li atoms in a trap with $\omega = 1$Hz (with coupling parameter $\xi = 1$, see text).

be restricted by $N \ll \omega_\perp/\omega$, which for realistic traps is of the order of a few hundred or thousand. Let us now turn to an estimate of the conditions to reach the strongly interacting limit. Since the interaction between the atoms has a range much smaller than the interparticle spacing, at low temperatures only collisions between particles with different spins are allowed by the exclusion principle. Therefore, all the relevant interactions are characterized by a single parameter, the scattering length a corresponding to inter-component interaction. The effective 1D interaction can thus be represented as a zero-range potential of the strength $g = 2\pi\hbar^2 a/m l_\perp^2$, where $l_\perp$ is the width of the ground state in the transverse direction $(a \ll l_\perp)$[3] and m is the mass of the gas particle. The interaction strength in a Luttinger liquid is then characterized by the dimensionless parameter $\xi = g/\pi\hbar v_F$, where $m v_F^2/2 = k_B T_F$ is the Fermi velocity (at trap center). Remarkably, in a trapped gas $\xi \sim (a/l_\perp)(\omega_\perp/\omega N)^{1/2}$ and thus can be tuned externally either by changing the transverse confinement, or by changing the scattering length by magnetic field. The ratio of the transverse and the longitudinal frequencies is quite large, we can easily reach the strong coupling limit $\xi \sim 1$, even in a dilute gas.

Below we will study spin-charge separation according to the schematic setup outlined in Fig. 1. We assume that

a short far off-resonant laser pulse is focused at the center of the harmonic trap with a two-component atomic Luttinger liquid, where depending on the laser parameters (e.g. light polarization) density and spin wave packets can be excited (Fig. 1a). Spin-charge separation manifests itself in different propagation speed of the spin and density wave packets (see Fig. 1b). This can be probed at a later time with a second short laser pulse. Spin dependent optical potentials can be generated by a laser tuned e.g. between fine structure levels of excited Alkali states acts on the ground state "spin" in a way equivalent to the external magnetic field interacting with the spin density and thus introduce a spin density perturbation. The goal of the following derivations is thus to (i) derive the frequencies of the spin and charge modes of atomic Luttinger liquid confined in a harmonic trap, and (ii) to discuss the wavepacket dynamics as superposition of these modes.

We analyze the properties of a trapped Luttinger liquid by combining Haldane's low energy hydrodynamic description [6] with a local density approximation. As was shown by Haldane, any homogeneous interacting quantum liquid in one dimension can be described by an affective hydrodynamic Hamiltonian, which completely describes the behavior at wavelengths much larger than the interparticle spacing. In the case of spin-1/2 fermions, the effective Hamiltonian reads

$$H = \sum_{\nu=\rho,s} \int dx \frac{\hbar v_\nu}{2} \left[K_\nu \Pi_\nu^2 + \frac{1}{K_\nu} (\partial_x \phi_\nu)^2 \right], \quad (1)$$

Here the index ν counts the spin (s) and the density (ρ) excitations. The phenomenological parameters K_ν and v_μ completely characterize the low energy physics. For exactly solvable models like the 1D lattice model, can be directly expressed in terms of the microscopic parameters of the theory [13]. The gradients of the phases $\partial \phi_\nu$ are the density and the spin density fluctuations respectively. The canonical momenta Π_ν conjugated to the phases ϕ_ν are related to the spin and the density currents $j_\nu = v_\nu K_\nu \Pi_\nu$. In a rotationally invariant Fermi-gas the quantity $K_s = 1$, so that only independent parameters are K_ρ, and $v_{s,\rho}$[6, 7].

As apparent from Eq. (1) a distinctive feature of the LM Hamiltonian is the complete separation of the spin and the "charge" degrees of freedom. In a spatially homogeneous gas the spin and the charge waves propagate at the velocities v_c and v_s respectively. This is the essence of the spin-charge separation phenomena. In our cold atoms system the charge and the spin waves are modeled by excitations of the total and the relative densities of the components.

The local density approach to model a trapped gas assumes that the size of the atom cloud $R \gg k_F^{-1}$, i.e. the size of the gas sample is much larger than the interparticle separation, consistent with $N \gg 1$. The variation of K_μ and v_μ is assumed to originate only from the spatial dependence of the gas density, $K_\mu[n] \to K_\mu[n(x)]$ and $v_\mu[n] \to v_\mu[n(x)]$. We will also assume that the numbers of particles of the both "spins" are the same (i.e. the total spin of the system is zero). Our goal is now to derive the excitation spectrum in the charge and the spin sector analytically in the weak and strong coupling limits, and study the intermediate regime using numerical techniques. As a first step, this requires the calculation of the ground state density distribution.

Within the local density approximation, the ground state of the system can be characterized using the Thomas-Fermi equilibrium condition:

$$\frac{dE(n)}{dn} = \mu - V(x), \quad (2)$$

Here $E(n)$ is the internal energy of the gas per unit length of the gas as a function of its (total) density, μ is the chemical potential, $V(x) = m\omega^2 x^2/2$ is the longitudinal external potential, and ω is the frequency of the longitudinal confinement. This equation is just the expression of the fact that the energy cost of adding a particle to the system equals to the chemical potential corrected by the local value of the external potential. Generally speaking, the external potentials acting on the two different "spin" components can be different. This feature can be used to generate offsets in the densities of the components and hence produce the spin and the density excitations with laser light (see below).

In preparation for the interacting case we consider first a free gas ground state ($g = 0$). The density of the gas at a given position x is related to the local value of Fermi momentum, $k_F(x) = \pi n(x)/2$. The internal energy of the gas is just the density of the kinetic energy (the so called quantum pressure) $E(n) = \hbar^2 \pi^2 n^3(x)/24m$. Substituting these expressions into Eq.(2) we find

$$n_{TF}(x) = n_0 \sqrt{1 - \frac{x^2}{R_{TF}^2}}, \quad (3)$$

for $|x| < R_{TF}$, and 0 otherwise. Here $n_0 \equiv n(x = 0) = (8\mu m/\hbar^2 \pi^2)^{1/2}$ is the density in the center and $R_{TF} = (2\mu/m\omega^2)^{1/2}$ is the Thomas-Fermi size of the cloud [14]. From the requirement that the integrated density equals the particle number we have the condition $\mu = \hbar \omega N/2$.

The excitations of the gas can be found from the equations of motion following from the Hamiltonian (1):

$$\dot{\phi}_\nu = K_\nu(x) v_\nu(x) \Pi_\nu, \ \dot{\Pi}_\nu = \frac{\partial}{\partial x} \frac{v_\nu(x)}{K_\nu(x)} \frac{\partial}{\partial x} \phi_\nu. \quad (4)$$

In particular, for an ideal gas we can use the density profile (3) and find the equations for the mode functions:

$$-\epsilon^2 \phi_\nu = \hbar^2 \omega^2 (1 - \tilde{x}^2)^{1/2} \frac{\partial}{\partial \tilde{x}} (1 - \tilde{x}^2)^{1/2} \frac{\partial}{\partial \tilde{x}} \phi_\nu,$$

where $\tilde{x} = x/R_{TF}$. The solution is given by $\phi_{\nu n} = A_\nu \sin(\epsilon_n \arccos \tilde{x}) + B_\nu \cos(\epsilon_n \arccos \tilde{x})$. The discrete spectrum of eigenfrequencies is found by analyzing the boundary conditions: $\epsilon_\nu(n) = \hbar\omega(n+1)$ [14] both for the

spin and the density modes. The first modes ($n = 0$) and the wavefunctions $\phi_\nu \sim x/\sqrt{1 - \tilde{x}^2}$ correspond to harmonic oscillations of the center of mass of the total density and the total spin (dipole modes).

Before starting with the perturbation theory in small interaction parameter $\xi \ll 1$, we note that in a finite system there is additional energy scale, which is the level spacing ($\hbar\omega$). In order for the interaction effects manifest themselves in a way similar to a bulk system, we need the interaction to be stronger than the level spacing, $n(x)g \gg \hbar\omega$, in contrast to the case of very weak interactions studied in [5].

In a homogeneous gas the Luttinger parameters in the Hamiltonian (1) to the lowest order in $\xi = g/\pi\hbar v_F \ll 1$ can be found using perturbation theory: $K_s = 1$, $v_s = v_F(1 - \xi/2)$, $K_\rho = 1 - \xi/2$ and $v_\rho = v_F(1 + \xi/2)$ [15]. The energy of the ground state can be obtained by averaging the interparticle interaction over the ground state, $E_0 = \hbar^2\pi^2 n^3/24m + gn^2/4$. In the spirit of the local density approximation we substitute a spatially dependent density $n(x)$ in the expressions for homogeneous gas. Then, using Eq.(2) we find that in the first order in $\xi = g/\pi v_F(0)$, the density of the gas uniformly decreases by $\delta n(x) = -2gm/\hbar^2\pi^2$, i.e. the interaction reduces the density, as expected. This simple conclusion holds everywhere as long as $g \ll \hbar^2 n(x)/m$, i.e.

$$(R_{TF} - x)/R_{TF} \lesssim (gm/\hbar^2 n_0)^2 \sim O(\xi^2) \qquad (5)$$

The velocities of the spin and the density waves are

$$v_{s,\rho}(x) = \frac{\pi\hbar n_{TF}(x)}{2m}(1 - \frac{A_{s,\rho}gm}{\pi^2\hbar^2 n_{TF}(x)}), \qquad (6)$$

where $A_s = 3$ and $A_\rho = 1$. Using the expansion in powers of ξ of the Luttinger parameters and the density profile (2) we find, that the frequency of the density dipole mode does not depend on the interaction (as it should be), while the the spin dipole mode shift is given by an integral logarithmically diverging at the border of the gas cloud. The divergence occurs due to localization of the excitations of a free gas close to the gas cloud border and arises in any potential, which is a power law in x. Using the condition (5) to cut off the divergence, we find

$$\delta\omega_{s1} = -\omega\frac{3gm}{\pi^2\hbar^2 n_{TF}(0)}\log\frac{\pi\hbar^2 n_{TF}(0)}{2gm}.$$

This shift is negative and can be observed by comparing the spin and the density oscillations of the gas cloud. Note that in a harmonic trap the perturbation theory requirement is stronger than in a homogeneous Luttinger liquid: we have to require $\xi\log(1/\xi) \ll 1$ instead of simply $\xi \ll 1$.

For higher modes the application of the perturbation theory in Eqs.(4) turns inconvenient and the frequencies of the excitations can be analyzed within the WKB approximation. The accuracy of the WKB spectrum estimation is $\sim 1/\pi^2 n^2$ [16], whereas the expected corrections are of order g/v_F. Therefore for sufficiently high
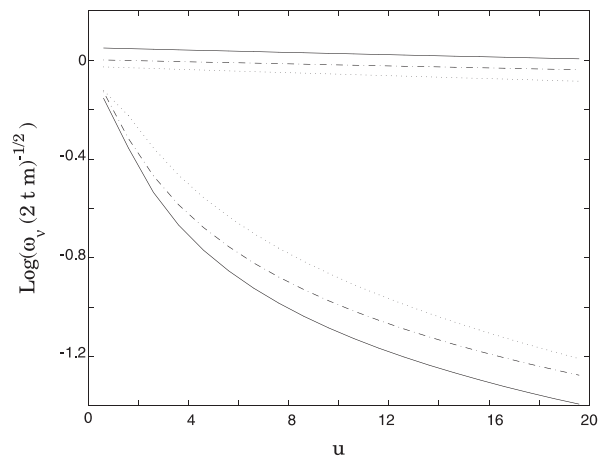

Fig. 3 The level spacing (in units of ω) between the spin (solid line) and the "charge" (dashed line) modes vs. $\log\xi$ for different central densities: $n(0) = 0.25$ (solid), $n(0) = 0.42$ (dot-dashed), $n(0) = 0.58$ (dotted).

n the eigenfrequencies can be reliably obtained from the WKB quantization condition[17],

$$\int_{-x_0}^{x_0} p(x)dx = \hbar\pi(n + \alpha), \qquad (7)$$

where $p(x)$ is the WKB momentum corresponding to a given energy, n is the (integer) quantum number, x_0 is the classical turning point and the constant $\alpha = 1$ is fixed by comparing the WKB results and the exact solutions of Eqs.(4) for a weakly interacting gas. Substituting the dispersion relation $\epsilon = v_{\rho,s}(x)p(x)$ with the velocities (6) into Eq.(7) we obtain the the same sort of logarithmically diverging integrals as those for in the perturbation theory above. By regularizing them using the condition (5) we find, that

$$\epsilon_{\rho,s} = \hbar\omega(n + 1)(1 - \frac{2gmA_{\rho,s}}{\pi^2\hbar^2 n_{TF}(0)}\log(\frac{\hbar^2 n_{TF}(0)\pi}{mg})). \qquad (8)$$

This simple WKB calculation confirms the interaction dependent split of the spin and the density oscillation frequencies.

In the limit of very large interaction strength ($g \gg \pi\hbar v_F$) the repulsion between the atoms of the two different species is very strong. Hence, the properties of the gas are similar to those of an ideal single component gas of indistinguishable particles. The density profile is still given by Eq.(3), but now with $n_0 = n_\infty = (2\mu m/\hbar^2\pi^2)^{1/2}$, $\mu = \hbar N\omega$, and $R_{TF} = R_\infty = (2\mu/m\omega^2)^{1/2}$. This distribution is less dense and thus broader than that for a weakly interacting gas. The density wave speed is equal to the Fermi velocity $v_F = \hbar\pi n_\infty/m$ and, after integration in Eq.(7), we find that spectrum of the density waves is the same as in the non-interacting case above. In turn, the relation between the energy and the WKB momentum for the spin wave is given by

$$\epsilon_s(p) = \frac{B\hbar n^2(x)p(x)}{m^2 g},$$

where the coefficient $B \sim 13$ (± 2) was found numerically. Once again, using the quantization condition (7) and cutting off the logarithmically divergent integral at the point $n \ll g$, we find, that

$$\epsilon_{ns} = \hbar\omega(n+\alpha)\frac{\hbar^2 B n_\infty}{gm\log(gm/\hbar^2 n_\infty)},$$

where $\alpha \sim 1$. As it is clear from the latter expression, the interaction profoundly changes the properties of the spin mode. In the limit of the strong interaction the level spacing decreases and is much smaller than that between the density waves (ω).

In order to confirm our analytical results, we performed a numerical calculation valid for arbitrary interaction strength based on a lattice model. Using the exact solution [18, 19] for calculation of the Luttinger constants and the Thomas-Fermi approximation (2), we determined the WKB level spacings for the spin and the charge modes. The results are presented in Fig. 2 as a plot of the excitations level spacing vs. the dimensionless interaction strength $g/\pi v_F$ calculated at the center of the trap.

As outlined in Fig. 1, wavepackets of the spin and density excitations can be generated by short off-resonant and state selective laser pulses focused to a spot size ℓ with $R \gg \ell \gg k_F^{-1}$, where R_{TF} is the size of the atom cloud and k_F^{-1} the interparticle distance. This procedure is analogous to the MIT setup originally used to study propagation of sound waves in elongated condensates [20]. Fig. 1b shows as an example the wave packet dynamics for the states $|F=1/2, M_F=\pm 1/2\rangle$ of 6Li with interaction parameter $\xi = 1$, corresponding to $N = 500$ particles at trap frequency of $\omega = 1$Hz, $\omega_\perp = 250$kHz and scattering length $a_s = 23\text{Å}$. Tuning near the Feshbach resonance (at $B = 800$G) allows an increase of the scattering length by one order magnitude, allowing for $N = 1000$ atoms at a trap frequency of $\omega_\perp = 100$kHz.

Let us finally now turn to a discussion of the life time of the excitations and temperature requirements. The Hamiltonian (1) represents only the first term in hydrodynamic expansion $q/k_F \ll 1$. The higher order terms

originate from, for example, non-linearity of the fermionic spectrum and mix the excitations with each other. The first corrections are of the third power in Π and $\partial\phi$, and hence lead to scattering of the excitations. To study the relaxation phenomena we switch for simplicity to a case of a single-component LM. According to Haldane, the generic term is given by $V_{int} = \gamma\hbar^2/m(\partial\phi)^3$, where $\gamma \sim 1$ [6]. The damping of the oscillations can be found from Fermi's Golden rule applied to the interaction Hamiltonian V_{int} and using the second quantized representation for the phonon field.A straightforward calculation gives $\Gamma_{T=0} \sim \hbar^2 q^4 L/m^2 u$. for $k_B T \ll \hbar\omega_q$, which is the decay rate in a process where a particle with the energy $\hbar\omega_q$ decays into a pair of particles with $\omega_1, \omega_2 < \omega_q$). Note that $\omega_q \gg \omega$, otherwise there are no final states for such decay instability (i.e. the lowest excitations are very stable at very low temperatures). In the case of $k_B T \gg \hbar\omega_q$(but still $T \ll T_F$) we find $\Gamma_T \sim q^2 L k_B^2 T^2/m^2 u^3$ which corresponds to Landau damping, i.e. the contribution of a process in which the damping occurs by scattering a high-frequency excitation with $\hbar\omega_q' \sim k_B T$. Both results contain the size of the sample L and are only valid in the collisionless regime $\Gamma/\omega_q \ll 1$. This is the case for sufficiently small temperatures (or high number of particles). Indeed, for the lowest excitations $Lmu/\hbar \sim N$, $\omega_q \sim \omega \sim \epsilon_F/\hbar N$, so that $\Gamma/\omega_q \sim (k_B T/\epsilon_F)^2 \ll 1$, i.e. the excitations are only weakly damped.

In conclusion we performed the analysis of a double component Fermi gas confined in a harmonic trap. Based on the LM we have investigated the nature of the excitations and analyzed an experiment where spin-charge separation can be observed "directly" in experiments addressing the spectral properties of the lowest excitations with laser light.

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